

Short-Message Communication and FIR System Identification using Huffman Sequences

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Abstract—Providing short-message communication and simultaneous channel estimation for sporadic and fast fading scenarios is a challenge for future wireless networks. In this work we propose a novel blind communication and deconvolution scheme by using Huffman sequences, which allows to solve three important tasks in one step: (i) determination of the transmit power (ii) identification of the discrete-time FIR channel by providing a maximum delay of less than $L/2$ and (iii) simultaneously communicating $L - 1$ bits of information. Our signal reconstruction uses a recent semi-definite program that can recover two unknown signals from their auto-correlations and cross-correlations. This convex algorithm is stable and operates fully deterministic without any further channel assumptions.

I. INTRODUCTION

Next generation wireless communication networks have to cope simultaneously with many different and partially contradicting tasks. It becomes increasingly apparent that current technologies will not be able to meet the emerging demands of future mobile communication systems, such as supporting sporadic and short-message traffic types for the internet of things, machine-type communication and sensor applications. In particular, once a node wakes up in a sporadic manner to deliver a message it has first to indicate its presence to the network. Secondly, training symbols (pilots) are used to provide sufficient information at the receiver for estimating link parameters such as the channel. Finally, after exchanging a certain amount of control information the device transmits its desired information message on pre-assigned resources. In current systems these steps are usually performed in separate communication phases yielding a tremendous overhead once the information message is sufficiently short and the nodes wake up in an unpredictable way. Therefore, a redesign and rethinking of several well-established system concepts and dimensioning of communication layers is necessary to support such traffic types in an efficient manner [1]. This has gained again deeper interest in methods for blind demixing and deconvolution methods which can operate on a short-frame basis. Disadvantages of the classical techniques hereby lie in its statistical flavour and in the lack of efficiency and robustness, since the algorithms for identification are often iterative and rarely have convergence guarantees.

In this work we will use a convex program for the channel and data reconstruction first introduced in [2] for the noise free case and show its numerical stability. The blind reconstruction

can hereby be re-casted as a phase retrieval problem with additional knowledge of the auto-correlations of the data and the channel at the receiver. The uniqueness of the phase retrieval problem can then be shown by constructing an explicit dual certificate in the noise free case, as was shown in [2] for almost all signals and channels. In [3] and more detailed in [4] we have shown, that the uniqueness derived in [2], holds indeed deterministically, given a particular co-prime condition is fulfilled. The latter condition was already shown in [5] to be necessary for blind deconvolution. Using therefore sequences with good autocorrelation properties allows to estimate the autocorrelation of the channel from the observations which in turn enables blind deconvolution by solving the corresponding phase-retrieval problem. Here, we propose to use Huffman sequences for this purpose which comes with further advantages from the system identification perspective. This scheme allows for simultaneous FIR channel estimation, transmit power estimation, to resolve near-far effects, and the communication of short messages.

The paper is organized as follows: First we motivate and introduce in Section II deterministic blind deconvolution with additional knowledge of autocorrelations for a short-message communication. Then, in Section III, we investigate signal classes of good and known autocorrelations, yielding to a codebook design of Huffman sequences. Due to the impulsive-equivalent behaviour of their autocorrelations, we show in Section IV, that they can be used to obtain a good estimate of the channel autocorrelation at the receiver. By using the SDP in [2] we show a perfect reconstruction of channel and data in the noise free case. Finally, in Section IV-B, we demonstrate numerically noise robustness of our reconstruction scheme in terms of bit-error-rates.

II. BLIND DECONVOLUTION

One-dimensional blind deconvolution problems occur in many signal processing applications, such as in digital communication over wired or wireless channels, where the channel is modeled as a linear time invariant (LTI) system, which has to be blindly identified or estimated.

$$y_k = (\mathbf{h} * \mathbf{x})_k = \sum_l h_l x_{k-l}. \quad (1)$$

If the receiver has some statistical knowledge of the LTI system, as given for example by second or higher order

statistics, known blind channel equalization and estimations were already developed in the 90's (see for example in [6–8]). If no statistical knowledge of the data \mathbf{x} and the channel \mathbf{h} is available, for example, for fast fading channels, one can still ask under which conditions on the data and the channel a blind channel or system identification is possible. Necessary and sufficient conditions in a multi-channel setup were first derived in [5] and continuously further developed (see e.g. [9] for a nice summary). However, the desired application here is a single-channel blind deconvolution of short signals (frame length is in the order of 1, 2, ... times the maximum delay spread of the channel), whereas previous methods often fail.

Recent progress in low-rank matrix recovery have put the one-shot blind deconvolution as a prototypical bilinear inverse problem back into focus. Using lifting, new results are obtained in [10] for randomized cyclic convolutions. There, the data signal lies in a random low-dimensional subspace and with certain incoherence assumptions it can be recovered with high probability using nuclear norm minimization. The computational aspects of the unlifted problem has been tackled recently in [11] with a clever initialization overcoming with high probability the non-convex nature such that gradient based algorithms will not stuck in a local minima. Although this renders blind deconvolution tractable in theoretical terms this (i) requires cyclic extensions which can be itself in the order of the signal length for our desired application, (ii) it requires common knowledge on random parameters which is often not feasible and (iii) even in the noiseless setting the recovery guarantees are probabilistic and it is therefore difficult to fulfill strict system requirements.

We will therefore address in this context blind (aperiodic) deconvolution again in the classical framework of polynomial factorization. Here, the convolution (1) transfers with the z -transform \mathcal{Z} , given for any $\mathbf{x} \in \mathbb{C}^L$ as

$$\mathcal{Z} : \mathbb{C}^L \rightarrow \mathbb{C}[z], \quad \mathbf{x} \mapsto (\mathcal{Z}\mathbf{x})(z) = X(z) := \sum_{k=0}^{L-1} x_k z^{-k}$$

to a polynomial multiplication

$$Y(z) = H(z) \cdot X(z) = \sum_{k=0}^{L+K-2} y_k z^{-k}. \quad (2)$$

Note, Y, H , and X are polynomials in the variable $z^{-1} \in \mathbb{C}$. Hence, given the observation \mathbf{y} and further constraints/knowledge of \mathbf{h} and \mathbf{x} , the recovery problem is equivalent to find the factorization (2). Indeed, the program in [2] obtains the right unique factorization by additionally knowledge of the autocorrelations $\mathbf{H}\mathbf{H}^*$ and $\mathbf{X}\mathbf{X}^*$ of the factors, which have to be co-prime.

III. GOOD AUTOCORRELATION SEQUENCES

Blind deconvolution using the SDP proposed in [2] and [3, 4] is based on the idea that one has access to the autocorrelations of the transmitted data signal and of the channel. For any $\mathbf{x} \in \mathbb{C}^L$ the autocorrelation $\mathbf{a}_x := \mathbf{x} * \overline{\mathbf{x}^-}$ is defined by the convolution of \mathbf{x} with its conjugate-time-reversal given by $(\overline{\mathbf{x}^-})_k = \overline{x_{L-1-k}}$ for $k \in \{0, \dots, L-1\}$. We will explain

this program later in Section IV. In the desired application, however, the receiver has only access to the observed channel output. But, if we a-priori fix the autocorrelation \mathbf{a}_x of the data, the autocorrelation of the channel \mathbf{a}_h can be estimated from the channel output and we can use the ambiguities of \mathbf{a}_x for communicating a short message in \mathbf{x} . Let us illustrate this in the noiseless setting. Using the WIENER-LEE relations:

$$\mathbf{a}_y = \mathbf{y} * \overline{\mathbf{y}^-} = (\mathbf{x} * \overline{\mathbf{x}^-}) * (\mathbf{h} * \overline{\mathbf{h}^-}) = \mathbf{a}_x * \mathbf{a}_h, \quad (3)$$

see e.g. [12, (2.29)], we can retrieve \mathbf{a}_x from \mathbf{a}_y if $|\mathbf{A}_x(z)| > 0$ on the unit circle. It is obvious that this is possible and sufficiently stable if the convolution behaves close to an identity for the desired channels (we will assume that the maximum delay spread is known). In other words, to obtain from the received signal $\mathbf{y} = \mathbf{x} * \mathbf{h}$ the autocorrelation of \mathbf{h} , we need further properties of \mathbf{x} , in the sense that the autocorrelation is close to an impulse.

A. Huffman Sequences

For the cyclic (periodic) autocorrelation of sequences (vectors) $\mathbf{x} \in \mathbb{C}^L$, having the impulse-vector¹ δ_0 as autocorrelation, are called *perfect sequences*, see e.g. [12, p. 5.8]. Unfortunately, for aperiodic autocorrelations $\mathbf{a} = \mathbf{a}_x = \mathbf{x} * \overline{\mathbf{x}^-}$, it is easily seen that a perfect aperiodic autocorrelation can not exist if $\mathbf{x} \neq \delta_0$, since for any $\mathbf{x} \in \mathbb{C}_{0,0}^L := \{\mathbf{x} \in \mathbb{C}^L \mid x_0 \neq 0 \neq x_{L-1}\}$ with² $L \geq 2$ we obtain for the first and last coefficient

$$a_0 = x_0 \overline{x_{L-1}} = \overline{x_{L-1} x_0} = \overline{a_{2L-2}} \neq 0. \quad (4)$$

Nevertheless, there exists a huge literature on constructing almost perfect aperiodic autocorrelation sequences, see for example [12, Cha.6]. Since our goal is to use them for identifying the channel autocorrelation we will need impulsive ones, i.e., where most of the sidelobes vanish. In fact, the best of such impulse like autocorrelations were found by HUFFMAN [13] and are given by

$$a_k = \begin{cases} -1 & k = 0 \\ 0 & k \in \{1, 2, \dots, L-2\} \\ E & k = L-1 \\ 0 & k \in \{L, L+1, \dots, 2L-3\} \\ -1 & k = 2L-2 \end{cases} \quad (5)$$

with energy $E = \|\mathbf{x}\|^2 \geq |x_0|^2 + |x_{L-1}|^2 \geq 2$. The construction of such sequences is straight-forward in the z -domain by determining its zero. Following the lines in [14] we get for the autocorrelation in the z -domain

$$A(z) = \mathbf{A}_x(z) = -1 + Ez^{-(L-1)} - z^{-2(L-1)}, \quad (6)$$

which is a polynomial of order $2L-2$, having $2L-2$ zeros $\zeta \in \mathbb{C}$. Indeed, if $\zeta^{-L+1} = r > 0$ then ζ is a zero of $A(z)$ for

$$r_{\pm} = \frac{E}{2} \pm \sqrt{\frac{E^2}{4} - 1}, \quad (7)$$

¹ Given by the Kronecker tensor $(\delta_0)_k = \delta_{0,k} = 0$ for $k \neq 0$ and $\delta_{0,0} = 1$.

² If $L = 1$ we get the trivial multiplication $a = x\overline{x} = |x|^2 = 1$, having only a global phase solution $e^{i\phi}$.

which implies $r_+ = r_-^{-1} \geq 1$. Hence, the $2L - 2$ zeros have radius $R_{\pm} := r_{\pm}^{1/(L-1)}$ and are given for $k \in \{1, \dots, L-1\}$ by

$$\zeta_k^{\pm} := e^{-2\pi i \frac{k-1}{L-1}} \cdot R_{\pm}. \quad (8)$$

Since $\zeta_k^+ = 1/\zeta_k^-$, the $2L - 2$ zeros occur in conjugated-pairs, where $L - 1$ zeros lie on the circle of radius R_+ and the other $L - 1$ on the circle of radius R_- , i.e.,

$$A(z) = - \prod_{k=1}^{L-1} (z^{-1} - \zeta_k^-) \cdot \prod_{k=1}^{L-1} (z^{-1} - \zeta_k^+) = X(z)X(z)^*. \quad (9)$$

Note, we have to set the unit (scaling) to -1 , since that the last coefficient becomes -1 and the first $-\prod_{k=1}^{L-1} e^{4\pi k/(L-1)} = -1$, if we calculate the product in (9). By swapping the primes, i.e., the zeros, we can obtain 2^{L-1} different factorizations, the maximal amount of non-trivial ambiguities of the autocorrelation [4], yielding by the inverse z -transform to 2^{L-1} different Huffman sequences $\mathcal{Z}^{-1}X = \mathbf{x} \in \mathbb{C}_{0,0}^L$ having all the same energy E and autocorrelation (5). Since X is up to a unit $e^{i\theta}$ defined by its zeros, we can set the unit to 1 which yields as first coefficient $x_0 = 1$ and hence as last coefficient $x_{L-1} = -1$, see Figure 2. But if we assign $B_+ \subseteq \{1, \dots, L-1\}$ with $|B_+| \neq (L-1)/2$ (always true if L is even) zeros ζ_k^+ for X , then we have to assign $|B_-| = L-1-|B_+|$ zeros of radius R_- with $B_- = B_+^c$, which gives $\prod_{k \in B_+} \zeta_k^- \prod_{k \in B_-} \zeta_k^+ = R_+^{|B_-|-|B_+|} =: c^2 \neq 1$. Hence, we have to scale our selection by

$$X(z) := -c \prod_{k \in B_+} (z^{-1} - \zeta_k^-) \prod_{k \in B_-} (z^{-1} - \zeta_k^+), \quad (10)$$

where we assume L even, yielding always a positive scaling factor³ and therefore to $x_0 > 0$ and $x_{L-1} < 0$. Then indeed, forming the involution of X gives

$$\begin{aligned} X^*(z) &:= -z^{-L} \overline{X(1/\bar{z})} = -c \prod_{k \in B_+} (1 - \zeta_k^- z^{-1}) \prod_{k \in B_-} (1 - \zeta_k^+ z^{-1}) \\ &= c \prod_{k \in B_+} \zeta_k^+ \prod_{k \in B_-} \zeta_k^- \prod_{k \in B_+} (z^{-1} - \zeta_k^-) \prod_{k \in B_-} (z^{-1} - \zeta_k^+) \end{aligned}$$

and hence $X(z)X^*(z) = A(z)$.

Encoding rule: We have designed above a non-binary block code $\mathcal{C}_{E,L}$ of complex Huffman sequences $\mathbf{x} = \mathcal{Z}^{-1}X$ of length L and cardinality 2^{L-1} . This allows encoding of $L-1$ bits by its zeros (8). For the k th bit b_k we set then

$$\zeta_k^{\pm} = \begin{cases} e^{-2\pi i \frac{k-1}{L-1}} R_+ & , b_k = 1 \\ e^{-2\pi i \frac{k-1}{L-1}} R_- & , b_k = 0 \end{cases}. \quad (11)$$

Such a rule needs to be implemented efficiently. For example, by adjusting the phase in c and the main-sidelobe ratio, integer sequences can be generated recursively for certain lengths L [12].

Comments on the Peak-to-Average Power Ratio (PAPR): One drawback in using Huffman sequences without further restriction, i.e., with the autocorrelation given in (5), is that this comes with an PAPR

$$\text{PAPR}_{\mathcal{C}_{E,L}} := \max_{\mathbf{x} \in \mathcal{C}_{E,L}} L_1 \cdot \frac{\|\mathbf{x}\|_{\infty}^2}{\|\mathbf{x}\|_2^2} \leq L_1, \quad (12)$$

³The product is real-valued since $\prod_{k=1}^{L-1} e^{-2\pi i (k-1)/(L-1)} = e^{2\pi i (\sum_{k=1}^{L-1} k)/(L-1)} = e^{\pi i L} = 1$ if L is even and -1 if L is odd.

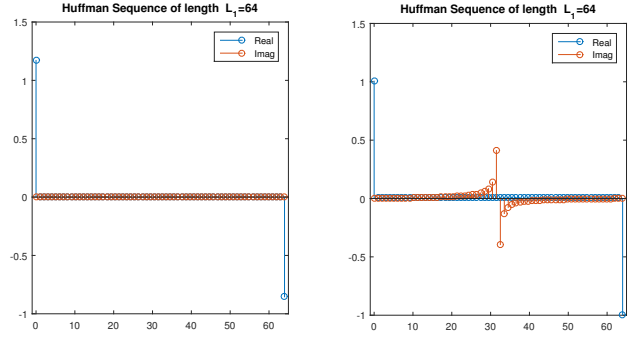


Figure 1. Huffman codeword in time for all one bit, $L = 64$ and $E = 2.1$. Figure 2. Huffman codeword in time for same amount of ones and zeros.

where for $E \gg 2$ the maximum is achieved for the all zero or all one bit codewords, having they energy located at the first and last coefficient, see Figure 1. If we set $E = 2$ we obtain the best possible PAPR of $L/2$, but loosing our code structure since $R^+ = R^- = 1$. Exemplary, by choosing $E = 2.1$ we obtain for $L = 128$ an PAPR of 19dB, which is slightly higher than for OFDM. Reducing the signal length to 64 only yields to 16dB, see Figure 1 and 2. To further reduce the PAPR extreme signals from the code have to be excluded, i.e., the one where the zeros are concentrated on one of the two circles.

IV. BLIND DECONVOLUTION AND DECODING OF HUFFMAN SEQUENCES

Let us assume we have a finite impulse response (FIR) channel $\mathbf{h} \in \mathbb{C}_{0,0}^K$ of length $K \geq 1$ with non-vanishing first and last coefficients. If we use Huffman sequences $\mathbf{x} \in \mathbb{C}_{0,0}^L$ with length $L = 2K + M$ for some $M \geq 0$, we receive

$$\mathbf{y} := \mathbf{x} * \mathbf{h} \in \mathbb{C}^{L+K-1}. \quad (13)$$

To apply Theorem 1 we need knowledge of the autocorrelation of \mathbf{x} and of \mathbf{h} . Indeed, using the WIENER-LEE relations (3), we get from the autocorrelation of the received signal

$$\mathbf{a}_y = \mathbf{y} * \mathbf{y}^- = \mathbf{x} * \mathbf{x}^- * \mathbf{h} * \mathbf{h}^- = \mathbf{a}_x * \mathbf{a}_h \in \mathbb{C}^{2L+2K-3}. \quad (14)$$

Using the property (5) of the Huffman sequences

$$\mathbf{a}_y = \mathbf{a}_x * \mathbf{a}_h = \begin{pmatrix} -1 \\ \mathbf{0}_{L-2} \\ E \\ \mathbf{0}_{L-2} \\ -1 \end{pmatrix} * \mathbf{a}_h = \begin{pmatrix} -\mathbf{a}_h \\ \mathbf{0}_M \\ E\mathbf{a}_h \\ \mathbf{0}_M \\ -\mathbf{a}_h \end{pmatrix} = \begin{pmatrix} \mathbf{a}_{y,1} \\ \mathbf{0}_M \\ \mathbf{a}_{y,3} \\ \mathbf{0}_M \\ \mathbf{a}_{y,5} \end{pmatrix}, \quad (15)$$

we can determine the channel autocorrelation by $\mathbf{a}_h = -\mathbf{a}_{y,1}$. Moreover, we can obtain from the ratio

$$E = \|\mathbf{a}_{y,3}\|_2 / \|\mathbf{a}_{y,1}\|_2 \quad (16)$$

the energy E of the Huffman sequences. Hence, we have determined \mathbf{a}_x and \mathbf{a}_y exactly. Inserting both autocorrelations in (23) yields up to a global phase ϕ the reconstruction

$$\mathbf{x}^\# = e^{i\phi} \begin{pmatrix} \mathbf{x} \\ \mathbf{h} \end{pmatrix} \quad (17)$$

and therefore the Huffman sequence $e^{i\phi}\mathbf{x}$. Since for even L , the first coefficient $x_0 > 0$, we just have to divide $\mathbf{x}^\#$ by the phase of $x_0^\#$ and obtain the original \mathbf{x} and \mathbf{h} . For L odd $x_0 < 0$.

Decoding Rule: From the estimated codeword $\mathbf{x}^\#$ we have to reverse (11) to obtain the zeros $\zeta_n^\#$, from which we calculate its absolute value $R_n^\#$ and phase $\phi_n^\#$. Then, for $k := (L-1)\phi_n^\#/2\pi \bmod L-1$ we set for the k th bit

$$b_k = \begin{cases} 1 & , R_n^\# \geq 1 \\ 0 & , R_n^\# < 1 \end{cases} \quad (18)$$

However, operationally this needs some further investigations for efficient sequence detection.

A. Deconvolution via Semi-Definite Programming

It is known, that the (aperiodic) *autocorrelation* of a vector signal $\mathbf{x} \in \mathbb{C}^N$ does not contain enough information to obtain a unique recovery, see for example [4] or [15], the idea is to use cross-correlation informations of the signal by partitioning \mathbf{x} in two disjoint signals $\mathbf{x}_1 \in \mathbb{C}^{L_1}$ and $\mathbf{x}_2 \in \mathbb{C}^{L_2}$ with $N = L_1 + L_2$, yielding $\mathbf{x}^T = (\mathbf{x}_1^T, \mathbf{x}_2^T)$ if stacked together. To obtain \mathbf{x} is equivalent to solve a phase retrieval problem via a semi-definite program (SDP). Here, the autocorrelation or equivalent the Fourier magnitude-measurements are represented as linear mappings on positive-semidefinite rank-1 matrices, see [2],[16]. This is known as *lifting*. The above partitioning of \mathbf{x} yields then a block structure for the positive-semidefinite matrix $\mathbf{x}\mathbf{x}^*$. The linear measurement \mathcal{A} is given component-wise by the inner products with the sensing matrices $\mathbf{A}_{i,j,k}$, defined below, which correspond to the k th correlation component of \mathbf{x}_i and \mathbf{x}_j for $i, j \in \{1, 2\}$. Hence autocorrelation and cross-correlation can be obtain from the same object $\mathbf{x}\mathbf{x}^*$. Let us define the $N \times N$ down-shift and $N \times L$ embedding matrix as

$$\mathbf{T}_N = \begin{pmatrix} \mathbf{0}_{N-1}^T & 0 \\ \mathbb{1}_{N-1, N-1} & \mathbf{0}_{N-1} \end{pmatrix} \quad \text{and} \quad \mathbf{\Pi}_{N,L} = \begin{pmatrix} \mathbb{1}_{L,L} \\ \mathbf{0}_{N-L,L} \end{pmatrix}. \quad (19)$$

Then, the $L_i \times L_j$ rectangular shift matrices are defined as

$$(\mathbf{T}_{L_j, L_i}^{(k)})^T := \mathbf{\Pi}_{N, L_i}^T \mathbf{T}_N^{k-L_j+1} \mathbf{\Pi}_{N, L_j}, \quad (20)$$

for $k \in \{0, \dots, L_i + L_j - 2\} =: [L_i + L_j - 1]$, where we set $\mathbf{T}_N^l := (\mathbf{T}_N^{-l})^T$ if $l < 0$. Then, the *correlation* between vectors of dimensions L_i and L_j is given component-wise as

$$(\mathbf{a}_{i,j})_k = (\mathbf{x}_i * \overline{\mathbf{x}_j})_k = \langle \mathbf{x}_i, (\mathbf{T}_{L_j, L_i}^{(k)})^T \mathbf{x}_j \rangle = \text{tr}(\mathbf{T}_{L_j, L_i}^{(k)} \mathbf{x}_i \mathbf{x}_j^*).$$

Hence this defines the linear maps $\mathcal{A}_{i,j,k}(\mathbf{X}) := \text{tr}(\mathbf{A}_{i,j,k} \mathbf{X})$, where $\mathbf{A}_{i,j,k}$ are the sensing matrices given as the correspondingly zero-padded $\mathbf{T}_{L_j, L_i}^{(k)}$ in (20). Stacking all the $\mathcal{A}_{i,j}$ together gives finally the measurement map \mathcal{A} . Hence, the $4N-4$ complex-valued linear measurements are given by

$$\mathbf{b} = \mathcal{A}(\mathbf{x}\mathbf{x}^*) = \mathcal{A} \begin{pmatrix} \mathbf{x}_1 \mathbf{x}_1^* & \mathbf{x}_1 \mathbf{x}_2^* \\ \mathbf{x}_2 \mathbf{x}_1^* & \mathbf{x}_2 \mathbf{x}_2^* \end{pmatrix} = \begin{pmatrix} \mathbf{x}_1 * \overline{\mathbf{x}_1} \\ \mathbf{x}_2 * \overline{\mathbf{x}_2} \\ \mathbf{x}_1 * \overline{\mathbf{x}_2} \\ \mathbf{x}_2 * \overline{\mathbf{x}_1} \end{pmatrix} = \begin{pmatrix} \mathbf{a}_{1,1} \\ \mathbf{a}_{2,2} \\ \mathbf{a}_{1,2} \\ \mathbf{a}_{2,1} \end{pmatrix}. \quad (21)$$

Note, \mathbf{b} is **not** an autocorrelation, but contains the part of the autocorrelation

$$\begin{pmatrix} \mathbf{x}_1 \\ \mathbf{0}_{L-1} \\ \mathbf{x}_2 \end{pmatrix} * \overline{\begin{pmatrix} \mathbf{x}_1 \\ \mathbf{0}_{L-1} \\ \mathbf{x}_2 \end{pmatrix}} = \begin{pmatrix} \mathbf{x}_1 * \overline{\mathbf{x}_1} + \mathbf{x}_2 * \overline{\mathbf{x}_2} \\ \mathbf{x}_2 * \overline{\mathbf{x}_1} \end{pmatrix}, \quad (22)$$

where we assumed for simplicity $L = L_1 = L_2$. Exactly this separation of the autocorrelation sum in (21) is an sufficient structure for semi-definite relaxations to solve the phase retrieval problem or equivalently the blind deconvolution problem. Note, since the cross-correlation $\mathbf{a}_{1,2}$ is the conjugate-time-reversal of $\mathbf{a}_{2,1}$, we only need $3N-3$ correlation measurements to determine \mathbf{b} . In [4],[3] we showed the following reconstruction algorithm:

Theorem 1. Let $\mathbf{x}_1 \in \mathbb{C}_{0,0}^{L_1}$ and $\mathbf{x}_2 \in \mathbb{C}_{0,0}^{L_2}$ such that the z -transforms $X_1(z)$ and $X_2(z)$ do not have any common factors. Then $\mathbf{x}^T = (\mathbf{x}_1^T, \mathbf{x}_2^T) \in \mathbb{C}^N$ with $N = L_1 + L_2$ can be recovered uniquely up to global phase from the measurement $\mathbf{b} \in \mathbb{C}^{4N-4}$ defined in (21) by solving the convex program

$$\text{find } \mathbf{X} \in \mathbb{C}^{N \times N} \quad \text{s.t.} \quad \begin{aligned} \mathcal{A}(\mathbf{X}) &= \mathbf{b} \\ \mathbf{X} &\geq 0 \end{aligned} \quad (23)$$

which has $\mathbf{X}^\# = \mathbf{x}\mathbf{x}^*$ as the unique solution.

This result can be easily reformulated as a blind-deconvolution program by knowledge of their autocorrelations. Therefore, we only have to identify with $\mathbf{x}_2 = \overline{\mathbf{h}^-}$ the conjugate-time-reversal of the *FIR channel* and with $\mathbf{x}_1 = \mathbf{x}$ the *data signal*. Then the measurements are

$$\mathbf{b} := \mathcal{A}(\mathbf{X}^\#) = \begin{pmatrix} \mathbf{x} * \overline{\mathbf{x}^-} \\ \mathbf{h} * \overline{\mathbf{h}^-} \\ \mathbf{x} * \overline{\mathbf{h}^-} \\ \overline{\mathbf{h}^-} * \overline{\mathbf{x}^-} \end{pmatrix} = \begin{pmatrix} \mathbf{a}_x \\ \mathbf{a}_y \\ \mathbf{y} \\ \overline{\mathbf{y}^-} \end{pmatrix} = \begin{pmatrix} \mathbf{a}_{1,1} \\ \mathbf{a}_{2,2} \\ \mathbf{a}_{1,2} \\ \mathbf{a}_{2,1} \end{pmatrix}. \quad (24)$$

Hence, inserting $\mathbf{a}_x, \mathbf{a}_y$ and \mathbf{y} in the algorithm (23) yields the solution $\mathbf{x}^\# = e^{i\phi}(\mathbf{x}, \overline{\mathbf{h}^-})$ as in (17), if $\mathbf{x} \in \mathbb{C}_{0,0}^{L_1}$ and $\mathbf{h} \in \mathbb{C}_{0,0}^{L_2}$ generate co-prime z -transforms $X(z)$ and $H^*(z)$. Since we have only finite many fixed \mathbf{X} inputs, but randomly chosen \mathbf{H} , the probability that \mathbf{H} share a common zero with \mathbf{X} is zero.

B. Simulation and Robustness

In practice we obtain only a noisy received signal

$$\mathbf{r} = \mathbf{y} + \mathbf{n} \quad (25)$$

disturbed by noise vector $\mathbf{n} \in \mathbb{C}^{L+K-1}$. From (15) we obtain

$$\mathbf{a}_r = \begin{pmatrix} -1 \\ \mathbf{0}_{L-2} \\ E \\ \mathbf{0}_{L-2} \\ -1 \end{pmatrix} * \mathbf{a}_h + \tilde{\mathbf{n}} = \begin{pmatrix} -\mathbf{a}_h \\ \mathbf{0}_M \\ E\mathbf{a}_h \\ \mathbf{0}_M \\ -\mathbf{a}_h \end{pmatrix} + \tilde{\mathbf{n}} = \begin{pmatrix} \mathbf{a}_{r,1} \\ \tilde{\mathbf{n}}_2 \\ \mathbf{a}_{r,3} \\ \tilde{\mathbf{n}}_4 \\ \mathbf{a}_{r,5} \end{pmatrix}, \quad (26)$$

where $\tilde{\mathbf{n}}$ is the correlated noise with \mathbf{y} and \mathbf{n} . To obtain a better result for the estimation of \mathbf{a}_h we use

$$\mathbf{a}_{\tilde{h}} = (\mathbf{a}_{r,1} + \mathbf{a}_{r,5})/2 = (\mathbf{a}_{r,1} + \overline{\mathbf{a}_{r,1}}^-)/2, \quad (27)$$

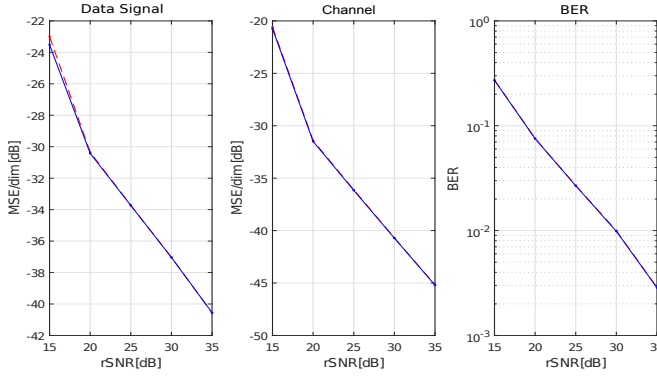


Figure 3. MSE for 13000 runs with $L = 32$ and $K = 8$ of data and channel reconstruction, and BER over rSNR. Red-dashed curve is with unknown and blue-solid with known energy.

which obtains a conjugate-symmetric vector, since \mathbf{a}_r is conjugate-symmetric. The only estimation parameter of \mathbf{a}_x is its energy, such that with (15) \mathbf{a}_x is the autocorrelation of a Huffman sequence $\tilde{\mathbf{x}}$ with Energy $\tilde{E} \geq 2$. Note, E is actually the main-sidelobe ratio, which remains constant by scaling of \mathbf{x} . Hence, we will search for the least-square solutions in (23)

$$\mathbf{X}^\# := \underset{\mathbf{X} \geq 0}{\operatorname{argmin}} \|\mathbf{b} - \mathcal{A}(\mathbf{X})\|_2^2. \quad (28)$$

By applying an SVD on $\mathbf{X}^\#$ for the best rank-1 solution, obtains after phase reversion in (17) our estimated signal $\mathbf{x}^\#$. In Figure 3 we plotted the MSE per dimension over the received signal-to-noise-ratio (rSNR), which scales nearly linear in dB for the signal (data) and the channel. We also added simulation results in blue-solid if receiver knows the Energy E exactly, which only effects the reconstruction for low SNR. In the third plot we see the uncoded Bit-error-rate (BER) over rSNR, which yields a BER of 10^{-1} at 18.5dB and of 10^{-2} at 29dB. To obtain better BER one might restrict the Huffman codeword to a smaller code, by excluding the vectors which are more likely affected by the noise in time-domain.

V. CONCLUSION

Recovering short-messages, estimating the channel and the corresponding transmit power solely from the channel output is a challenging blind signal processing task. It is potentially relevant for next wireless technologies in the area of the internet of things and sensor communication. We proposed here a novel scheme, based on Huffman sequences, which indeed can provide all these tasks simultaneously in one step. It requires to solve a semidefinite program which in turn returns the channel vector, the power at which the device transmits and after a decoding step the raw information bits.

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